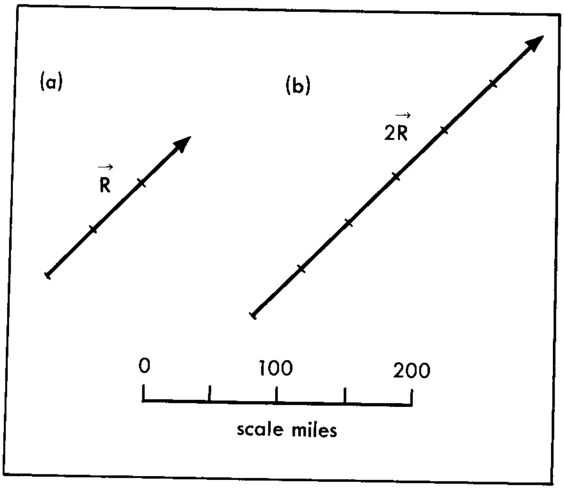
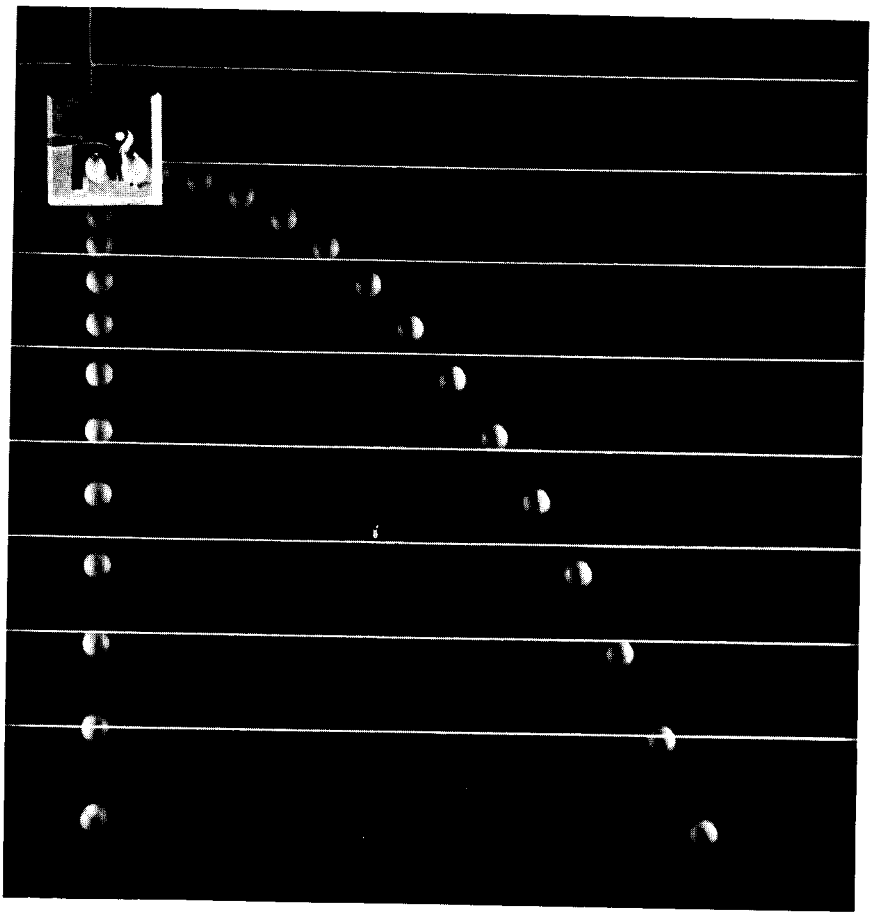


6–18. Displacement vectors for $\frac{1}{2}$ hour (a) and for two $\frac{1}{2}$ hours (b). The vector in (a) is obtained from that in Fig. 6–17 by multiplying $\frac{1}{2}$ hour by the velocity, a quantity with dimensions. The vector in (b) is obtained from that in (a) by multiplying by the pure number 2.

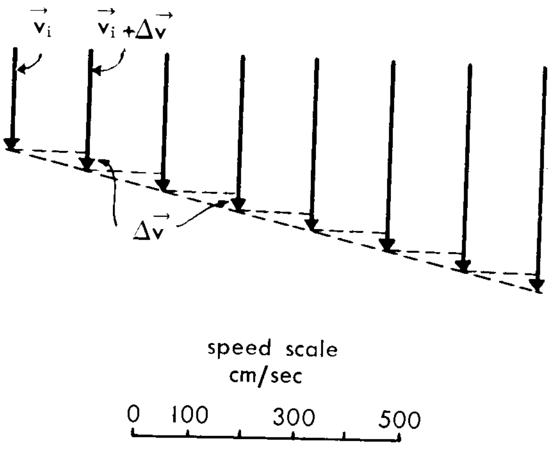
by multiplying the original displacement vector by the number 2. The total displacement of 300 miles northeast, as in Fig. 6–18 (b), can usefully be represented on the same vector diagram as the original displacement. It should not be placed on the same diagram as the velocity vector of 300 mi/hr because they are two different things, different physical quantities with different units.

6–5. Velocity Changes and Constant Vector Acceleration

Fig. 6–19 is a multiple-flash photograph of two balls. The ball at the left is falling straight down. Let us analyze its motion by finding the velocity vectors at successive intervals as the ball falls. We can get the average velocity vector for a given time interval by measuring the distance between two images of the ball and dividing by the time between the flashes which made those images. This gives us the length of the velocity vector; its direction is the direction of motion of the



6–19. A flash photograph of two golf balls released simultaneously from the mechanism shown. One of the balls was allowed to drop freely, and the other was projected horizontally with an initial velocity of 2 m/sec. The light flashes were $\frac{1}{30}$ of a second apart. The white lines in the figure are a series of parallel strings placed behind the golf balls, six inches apart. Why do the strings appear to be in the foreground?



6–20. The length of the arrows is equal to $2\frac{1}{2}$ times the displacements of the left-hand ball in Fig. 6–19 during the last eight successive intervals of $\frac{1}{30}$ of a second. Because we know the actual separations of the white lines in Fig. 6–19 and the time intervals, we can turn the magnitudes of these displacements into speeds. The scale enables you to read the lengths of the arrows directly as speeds in cm/sec.

ball from one image to the other. In Fig. 6–20 we have measured off the successive displacements and put them side by side. The white strings in Fig. 6–19 are 6 inches apart and the interval between flashes is $\frac{1}{30}$ of a second. Using these facts, we have computed a scale so that we can read these vectors directly as the average velocities.

A glance at Fig. 6–20 shows that the velocity vector changes steadily. In each successive interval it increases by the same amount. Consequently, we can find the velocity as $\vec{v}_n = \vec{v}_i + n\,\Delta\vec{v}$. Here $\vec{v}_i$ is the velocity vector with which we start. $\Delta\vec{v}$ is the constant change in velocity that occurs in each interval. By adding n of these changes to the original velocity, we get the velocity n intervals further along.

We can rewrite the last equation so that it more closely resembles the equations we developed for the description of motion along a preassigned path. (See Chapter 5, especially Sections 5–6 and 5–7.) There we defined $a = \Delta v/\Delta t$, that is, the acceleration along the path. Here by dividing $\Delta\vec{v}$ by Δt we shall introduce the vector acceleration $\vec{a} = \Delta\vec{v}/\Delta t$. Using it, our last equation becomes

$$\begin{aligned}\vec{v}_n &= \vec{v}_i + n\,\Delta t\,\frac{\Delta\vec{v}}{\Delta t}\\ &= \vec{v}_i + \vec{a}t.\end{aligned}$$

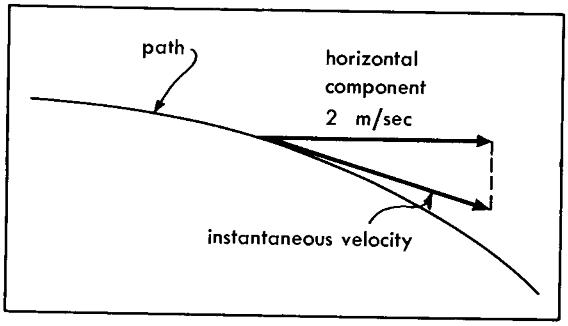
In the last line of this equation we have replaced $n\,\Delta t$ by t, the time during which the velocity has changed from its initial value $\vec{v}_i$ to its value in the nth time interval.

The velocity of the falling ball really increases steadily, as we can show by taking flash pictures at smaller and smaller time intervals. Therefore, after any time interval t it is $\vec{v}_t = \vec{v}_i + \vec{a}t$.

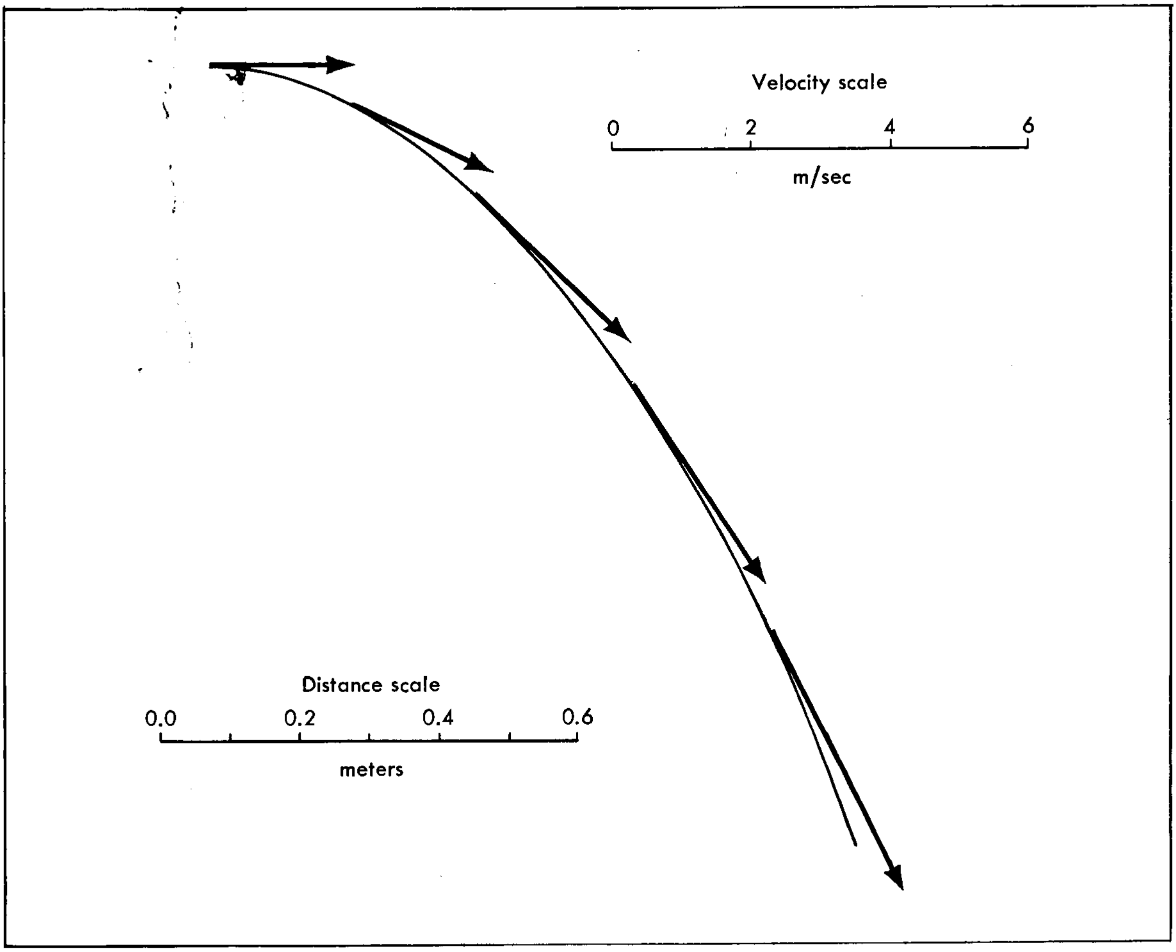
We have just described the motion of a falling ball in vector language. But since the ball moves on a predictable straight downward path, we hardly need the vectors. With speed and acceleration along the path we could have done the analysis equally easily. We would only need to add a statement that the motion is always straight down. The vector language, however, becomes far more useful when we analyze a more complicated motion. To see this let us get back to Fig. 6–19 and study the motion of the other ball, the one which moves out to the right in the figure.

The second ball in Fig. 6–19 moves both to the right and down. From the fact that the distance between the positions of the ball at successive flashes of the strobe light is greater for the later pictures, we see that the speed is increasing. Since the path is not a straight line, the direction of the velocity is changing too. We can analyze Fig. 6–19 to get the instantaneous velocity of the ball at various points along the path.* The

*One way of finding the instantaneous velocity in this case is to note that the horizontal component of the velocity is constant. This follows from the fact that the horizontal displacement is the same in each time interval. We then get $\vec{v}$ from this fact and the fact that the instantaneous velocity vector always points in the direction of the path. (See Fig. 6–21.) Other methods of analysis may give more precise results.



6–21. How to find the instantaneous velocity vector. It is tangent to the path and of such length that its horizontal component is equal to the initial horizontal velocity of the projectile.



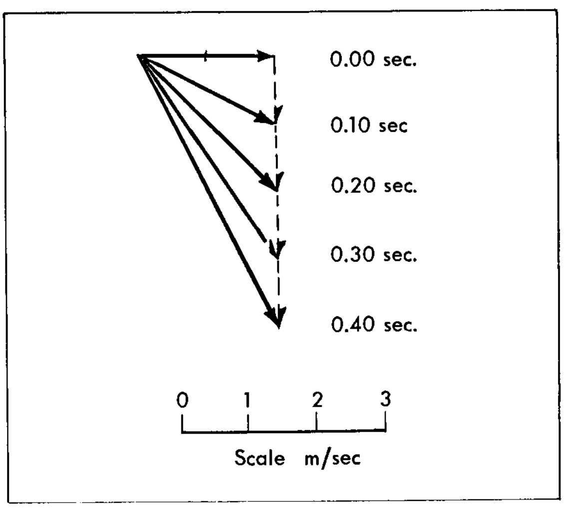
6–22. The position and velocity of the "thrown" golf ball in Fig. 6–19 are shown here on a single graph.

results of such an analysis are shown in Fig. 6–22. There both the position of the ball and its instantaneous velocity at 0.10-sec intervals are shown in the same graph. Note the two scales, one for distance and the other for velocity.

Fig. 6–23 shows only the sequence of velocity vectors of Fig. 6–22. Here, however, we have drawn the velocity vectors from the same starting point. Examination of this figure shows us that the successive vectors are obtained by adding a velocity vector of about 1 m/sec (actually it is 0.98 m/sec) directed vertically downward. We can express this rule in equation form. To do this we first express the components of the velocity. The horizontal component of velocity

$$\vec{v}_h = 2.00 \text{ m/sec, to the right}$$

stays constant throughout the flight. On the other hand, the vertical component is zero at



6–23. A sequence that shows only the velocity vectors of Fig. 6–22. Successive vectors are found by adding a constant vector directed vertically downward.

$t = 0.00$ sec and increases by 0.98 m/sec during each 0.10 sec. This is a uniform increase at the rate of 9.8 m/sec², so the vertical component of velocity at any time t is $\vec{v}_v = (9.8 \text{ m/sec}^2)t$, downward, where t is the time in seconds.

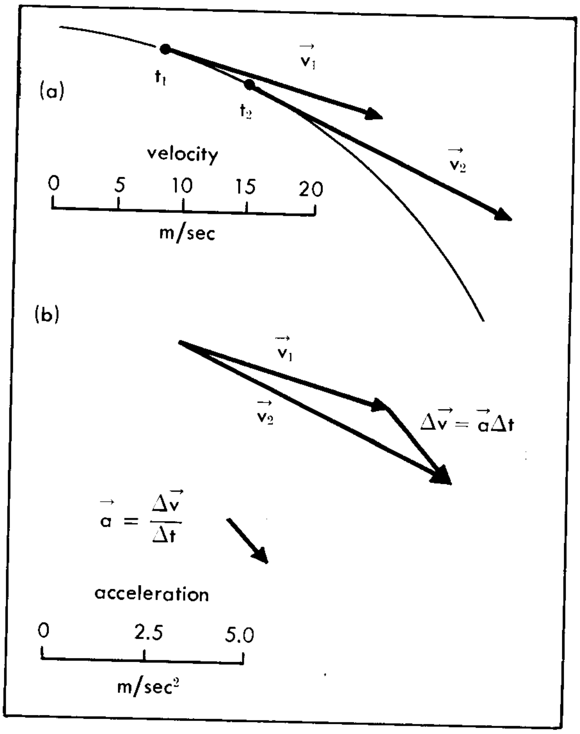
Now, combining the two rectangular components to give the vector velocity at any time t, we get

$$\vec{v}_t = (2.00 \text{ m/sec}), \text{ to the right} + (9.8 \text{ m/sec}^2)t, \text{ downward.}$$

The downward component of this vector is the product of a time and an acceleration. Since the time is measured in seconds and the acceleration in m/sec², the product has the units of m/sec, appropriate to a component of velocity. This is another illustration of the fact stated in Section 6–4, that the multiplication of a vector by a scalar gives a new vector, having the same direction, but of magnitude equal to the product of the scalar times the original vector.

We can put our equation for the motion of the ball thrown to the right into just the same form as we did for the ball falling straight down. Here (2.00 m/sec, to the right) is the initial velocity $\vec{v}_i$, and (9.8 m/sec², down) is the constant acceleration $\vec{a}$. So we get $\vec{v}_t = \vec{v}_i + \vec{a}t$ again. Notice that the acceleration of both balls is down. Also the downward motion is the same for both, as you can see by looking across the picture to check that one moves down the same amount as the other in each time interval. The only thing different in describing the motion of the two balls is the value of $\vec{v}_i$. The acceleration vector $\vec{a}$ is the same for both.

In Chapter 5 we found that $v_t = v_i + at$ described the speed at any time when there was constant acceleration along the path. The equation $\vec{v}_t = \vec{v}_i + \vec{a}t$ describes the velocity vector at any time as long as the acceleration vector is constant. Notice that the acceleration vector need not point along the path, as in the example of the right-hand ball in Fig. 6–19. Furthermore, $\vec{a}$ can be any constant vector of the right units. The 9.8 m/sec², down, which we found for the balls, is just a special value that occurs when balls move freely near the surface of the earth. In studying other motion we shall find other constant values of $\vec{a}$—and also acceleration vectors that change as time goes on. For instance, by pushing a ball we can get any $\vec{a}$ we wish.



6–24. To find the average acceleration in the interval $\Delta t = t_2 - t_1$: First find $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ and divide the vector difference by Δt. The result is the average acceleration vector $\vec{a}$, which can now be plotted with an appropriate scale as shown above.

6–6. Changing Acceleration and the Instantaneous Acceleration Vector

In the last section we described motion with constant vector acceleration. We introduced the vector acceleration to describe how the velocity vector changes. Even when the acceleration vector is not itself constant, we can introduce it in just the same way. We define it by $\vec{a} = \Delta\vec{v}/\Delta t$, where $\Delta\vec{v}$ is the vector change in v during the time interval Δt. Notice that this vector acceleration has the same direction as the change $\Delta\vec{v}$ of the velocity; since this change need not be in the same direction as $\vec{v}$, the acceleration $\vec{a}$ may point in any direction with respect to the motion. As we saw in the case of the right-hand ball in the last section, it need not be along the motion.

The vector acceleration we have just defined is the average acceleration over the time interval Δt. If the acceleration is itself changing as time goes on, $\vec{a}$ will depend on the time interval we choose. Let us take an example. Suppose a speedboat moves along the path shown in Fig. 6–24 (a).