MOTION ALONG A PATH

CHAPTER 5

A FREIGHT TRAIN is rolling down the track at 40 miles per hour. Around the bend a mile behind, a fast express appears, going at 70 miles per hour on the same track. The express engineer slams on his brakes. With the brakes set he needs two miles to stop. Will there be a crash? What we are called upon to do here is to predict where the two trains will be at subsequent times, and to find in particular whether they are ever at the same place at the same time. In a more general sense, we are asking about the connections between speeds, positions, and times.

The general subject of such relationships is called *kinematics*. In studying kinematics we do not concern ourselves with questions such as "Why does the express train need two miles to stop?" To answer such a question we would need to study in detail how the brakes slow down the train. Such questions as these will be considered in Part III on Mechanics. Here, we just consider the description of motion. We shall start with the discussion of motion along a given path without considering the position and direction of the path in space. Then in the next chapter we shall extend the discussion to describe the path.

In both of these chapters we shall draw on our ability to measure time and distance, for all motion is the changing of distance as time goes on. Usually we shall not think consciously of the time and distance measurements, but without them we would in fact be talking words without meaning.

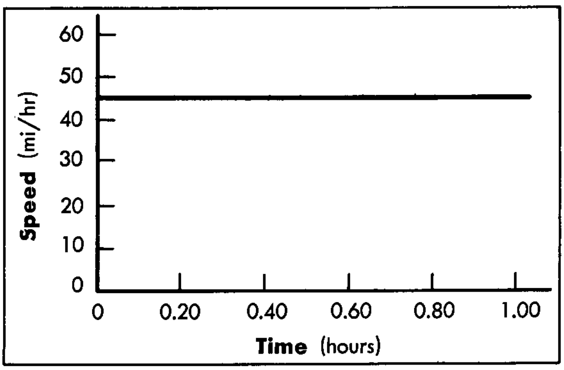
5–1. Speed and Distance

For a body moving with a constant speed, the relationship of time, speed, and distance is expressed simply. If we let d stand for the length of the trip, v for the speed, and t for the time needed for the trip, the equation

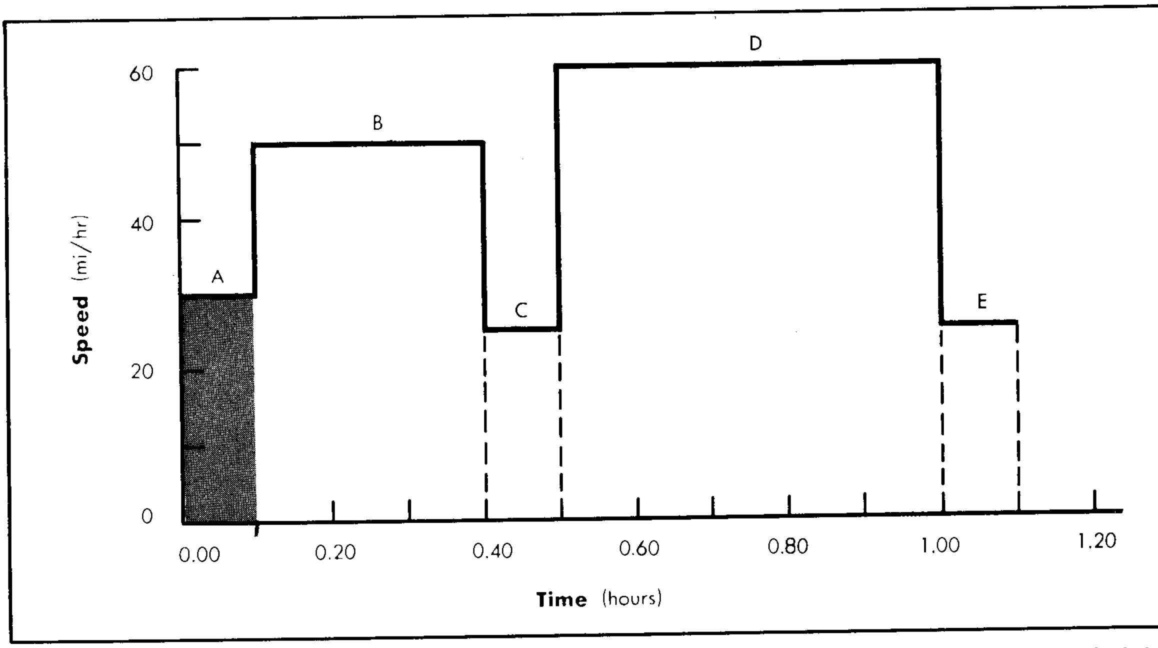
$$d = vt$$

relates these quantities for all cases of constant speed.

It is often convenient to use a graph to represent motions. Fig. 5–1 shows a graph of the speed versus time for a car which travels at 45 mi/hr. Taking some horizontal position on the graph, such as that corresponding to 0.20 hr, we find a reading on the vertical (or speed) axis of 45 mi/hr. In fact, we find 45 mi/hr for *any* time we select.



5–1. The speed of a car moving steadily may be graphed as a horizontal line.



5–2. The motion of a car moving at different speeds during different time intervals. The distance covered in any interval is measured by the area enclosed.

A more complicated motion is described in Table 1. To compute the distance traveled during the first time interval (0.10 hr long) we use the equation $d = vt$. The result is 3 miles. We can perform a similar calculation for each succeeding interval, and add the results to find that the total length of the trip is 53 miles.

Table 1

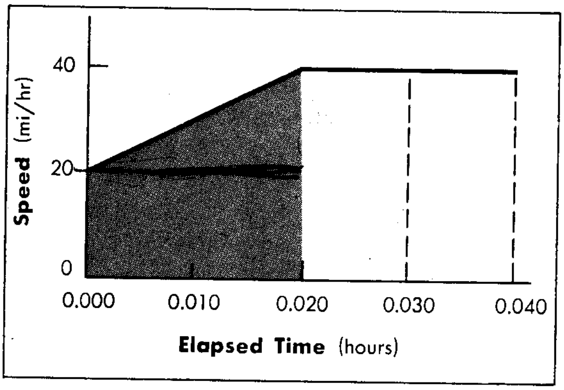
Motion of a Car at Variable Speed

Time interval number	Duration of interval	Speed during interval	Distance in miles
A	0.10 hr	30 mi/hr	3
B	0.30 hr	50 mi/hr	15
C	0.10 hr	25 mi/hr	2.5
D	0.50 hr	60 mi/hr	30
E	0.10 hr	25 mi/hr	2.5

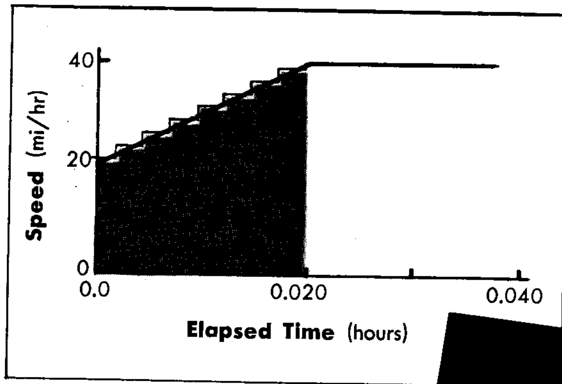
This motion is represented in Fig. 5–2. Actually, a real object could not move *exactly* according to this graph. Speed cannot increase in such sudden "jumps." However, a real car can make its changes of speed relatively rapidly. In that case, the graph of its motion will look very much like Fig. 5–2. We shall ignore the impossibility of sudden jumps in this discussion, so that we can keep our graph simple.

One great convenience of graphical presentation is that it enables us to see quickly when the car is going fast and when it is going slowly. Thus, the higher speeds occur "high" on the graph of speed versus time. Can the graph also tell us how far the car goes in each interval? The answer is "yes." Let us see how. During any one of the five intervals the car travels a distance given by the equation $d = vt$. In any interval, the height of the graph tells us the speed during the interval, and the horizontal length gives us the time. Thus v times t is the height times the base, or the "area" of the rectangle. This area is shaded for the first interval in Fig. 5–2. The units of these "areas" are different from the more common cm^2 or in^2 because one side of the rectangle is measured in hours, and the other is measured in mi/hr. The product in this case has units of hours $\times$ mi/hr = miles traveled.

The vertical axis of the graph represents the speed in mi/hr. But taking a ruler and actually measuring the vertical length in Fig. 5–2 to be 3.0 cm tells us nothing until we know that, for the particular scale of this graph, 3.0 cm represents 30 mi/hr. It is helpful to remember that the graph is a sort of scale drawing. Unlike a map which simply "scales down" distances, the graph



5–3. The speed-time graph for a car which is changing speed during part of its trip. Does the shaded area give the distance traveled during the time interval from 0.000 to 0.020 hours?



5–4. In this figure an imaginary car is altern[redacted] little faster and then a little slower than the car [redacted] that, eventually, it covers the same distance.

has different scales in the horizontal and vertical directions — scales which may differ not only numerically, but also in the nature of the physical quantities that they represent and therefore in their units. When we talk about the "height" being 30 mi/hr, we are using the graph in a way that gives the same answer no matter what scale we use in the actual drawing. For example, it makes no difference whether we use 0.5 cm or 1.0 cm to represent 10 mi/hr, but we must know which, and stick to it on any one graph.

Since heights and horizontal distances that we plot on the speed and the time scales of our graph are proportional to the actual speeds and times involved, any two areas on the graph are exactly proportional to actual distances the car moves. This fact often allows us to decide at a glance in which time interval the greatest distance is covered. For example, we can see that the area of the rectangle marked D in Fig. 5–2 is greater than that of any of the other rectangles. Therefore, we know without calculations that the car travels farther in the interval D than in any of the other intervals.

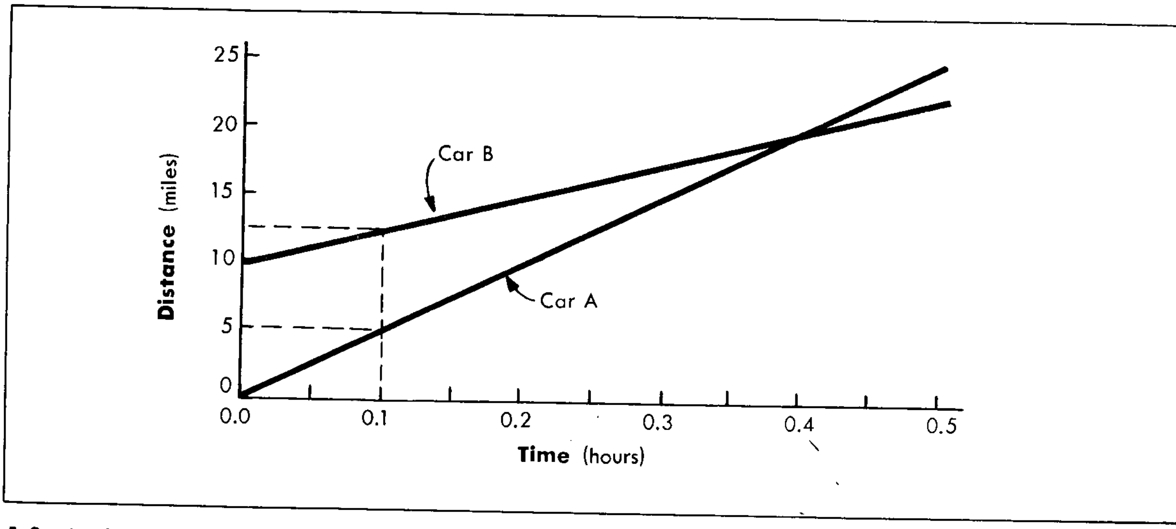
The total distance the car travels in 1.10 hr is obtained by adding up the "areas" of all the intervals in Fig. 5–2.

5–2. Varying Speeds

For the case we have considered, the graph did not give us any really new information because we had a method of computing distances without the aid of a graph. Now we shall use our graphical ideas to help analyze a more difficult problem.

Fig. 5–3 gives a graph of the speed of a car versus elapsed time. Can we tell how far the car goes in the first 0.020 hr? We can try to multiply the speed by the time, but we get into trouble, for we must now choose from a whole range of speeds. On the other hand, using the area under the graph, which worked as an alternate method for motion at constant speed, might also serve here, allowing us to solve graphically a problem that presents difficulties when tackled algebraically. Using the area to find our distance looks reasonable because we can approximate the sloping graph of Fig. 5–3 closely by the one in Fig. 5–4.

The graph of Fig. 5–4 represents the motion of an imaginary car that changes speed in steps (keeping constant speed during each step). Each step brings it to a speed a little greater than the speed of the real car at that instant. Then, while the imaginary car's speed remains constant, the speed of the real car gains on it and passes it. Next the imaginary car's speed increases by another step. The distance covered by the imaginary car is given by the area shaded under the stepped graph of Fig. 5–4. If we make the steps smaller, and more frequent, the two cars would never differ much in speed. Then the shaded area which gives the distance covered by the imaginary car would practically give the distance covered by the real car. And that shaded area, for many steps, is practically the shaded area under the graph of Fig. 5–3 for the real car. If you want to see another discussion which leads to a rigorous proof that the area under this speed-time graph gives the distance traveled, read the material in the box on the next page.

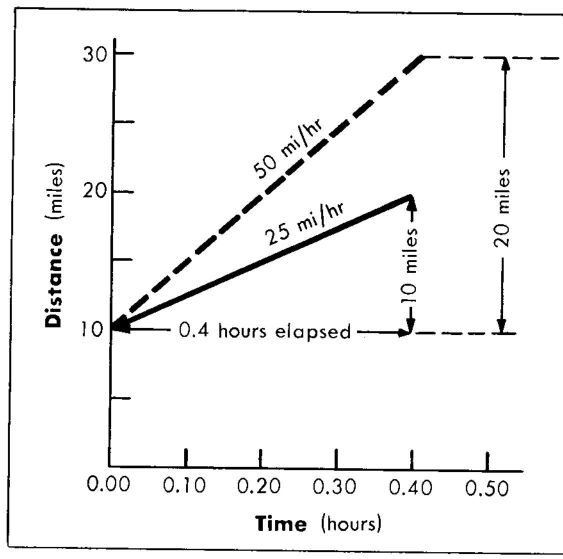


5–9. At what time will car A overtake car B?

From the graph we can calculate the speeds of the cars. In 0.1 hr, for example, car A goes from the position $d = 0$ to $d = 5$. It moves 5 mi and its speed is therefore 50 mi/hr. In the next 0.1 hr, it again goes 5 mi from $d = 5$ to $d = 10$; its speed is still 50 mi/hr. Because the graph is a straight line, the distance car A moves is the same for every 0.1 hr; therefore the speed of A is 50 mi/hr all the time. Car B also has constant speed. In each 0.1 hr it moves 2.5 mi, from $d = 10$ to $d = 12.5$ in the first 0.1 hr, from 12.5 to 15 mi in the second, and so on. Its speed is therefore 25 mi/hr.

In addition to the speeds, the graph tells us more. It says that car B starts 10 mi ahead of A, but A catches up. After 0.1 hr A is at $d = 5$ and B is at $d = 12.5$. A is therefore only 7.5 mi behind B. By the time 0.5 hr has passed we see that A is ahead of B. It is at $d = 25$, while B is only at $d = 22.5$. Just by looking at the graph we can tell how long it took A to catch up. At 0.4 hr both cars are at the same position, actually at $d = 20$; at 0.4 hr, therefore, A was just passing B.

In Section 5–1 we saw that on a graph of speed versus time we could tell at a glance at what times the speed was the greatest. The higher up the line occurred on the graph the greater the speed it represented. Now, however, we are dealing with a quite different graph — that of *distance* versus time. The speed is involved only indirectly in such graphs and is *not* shown by the height of the line above the time axis. For example, in Fig. 5–9 the line for car B is above that for car A in the entire interval from 0.00 hr to 0.40 hr, although car B is being overtaken during this interval and is certainly the slower of the two.



5–10. Higher speeds give steeper graphs of distance vs. time.

How can we tell from Fig. 5–9 which car is going faster? The answer is simple. One curve climbs more steeply than the other. For a given time interval, the steeper curve spans a greater interval of distance. Since the car which travels the greater distance in any given time is the faster, the faster car must be the one with the most steeply sloping graph. Car A is certainly going faster than car B. (That is why it passed B.)

Fig. 5–10 again illustrates the relationship between the steepness and the speed. The solid line is drawn for a car traveling at 25 mi/hr. We