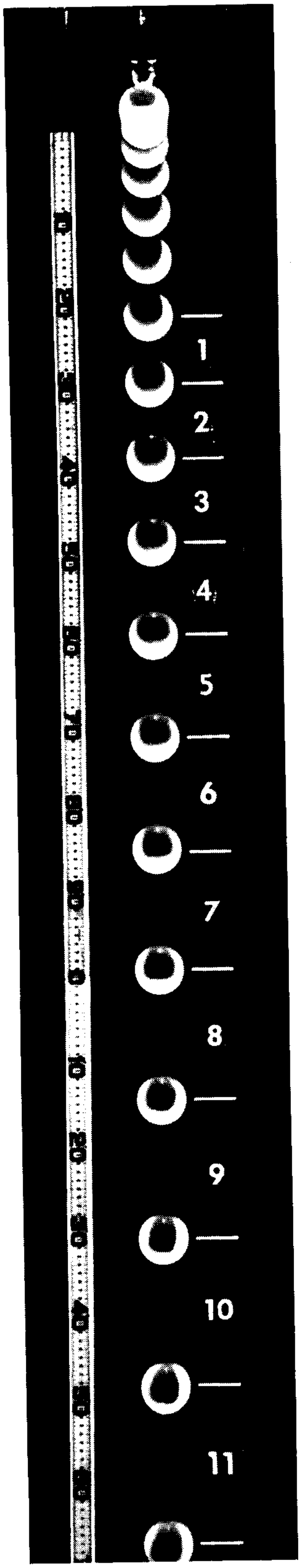
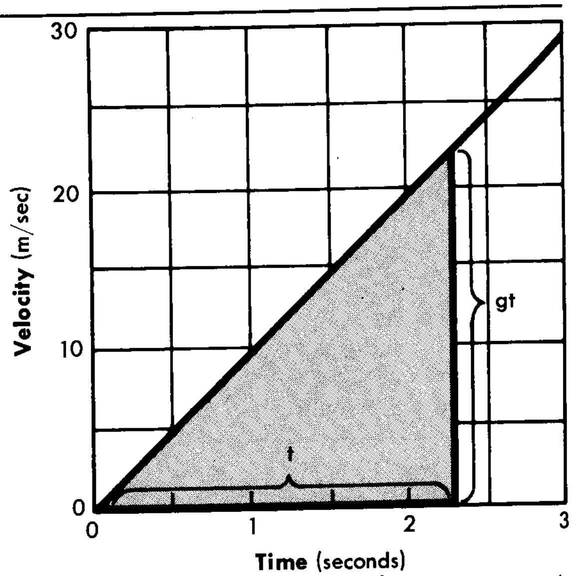


21–2. A flash photograph of a falling billiard ball. The distance scale is in centimeters, and the time interval between successive positions of the ball is $\frac{1}{30}$ second. This motion is analyzed in Table 2.

Table 2

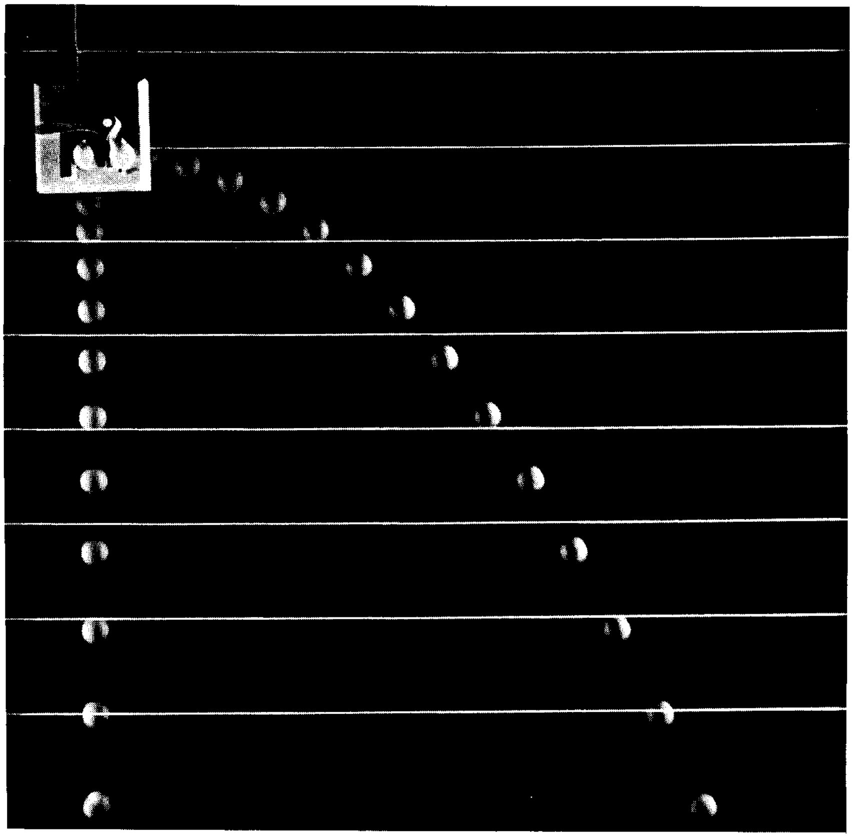
Interval Number	Interval Length Δx(cm)	Average Velocity $\Delta x/\Delta t = v$(cm/sec)	Change in Velocity Δv(cm/sec)	Acceleration $\Delta v/\Delta t$(m/sec^2)
1	7.70	231	32	9.6
2	8.75	263	31	9.3
3	9.80	294	32	9.6
4	10.85	326	34	10.2
5	11.99	360	33	9.9
6	13.09	393	32	9.6
7	14.18	425	32	9.6
8	15.22	457	32	9.6
9	16.31	489	35	10.5
10	17.45	524	32	9.6
11	18.52	556		
			Average acceleration	9.8

The analysis of the motion shown in Fig. 21–2. The calculated values of the acceleration are constant within the limits of accuracy of our measurements. Even though an accurate ruler was brought right up to the picture of the ball, the last significant figure in the Δx column is quite uncertain; just a reasonable estimate of a fraction of a millimeter. It has been retained, however, to reduce successive errors that would accumulate if we rounded off early. Notice that we have kept only three significant figures in the velocity column.



21–3. A velocity-time graph of a freely falling body. The distance fallen is given by the area under the curve — the area of the triangle with base t and height gt. This area is $\frac{1}{2}gt^2$.

We have limited our attention to compact, dense objects carefully selected to minimize the frictional resistance of the air. But if you drop a ping-pong ball it falls only a short distance before the force of air resistance balances the force of gravity, and the ball moves at constant speed. In general, air resistance becomes greater with higher speed. Therefore, if an object falls far enough, it will gain so much speed that the air resistance



21–4. A flash photograph of two golf balls, one projected horizontally at the same time that the other was dropped. The strings are 6 inches apart, and the interval between flashes was 1/30 second. This photograph is the same as Fig. 6–19.

becomes equal to the weight. Then the body continues to fall with constant speed. This final constant speed is called the terminal velocity of the falling body. (Problem: What happens if you throw a light bulb down faster than its terminal velocity?)

We can check that these deviations from free fall are indeed the result of air resistance by performing experiments in a vacuum. When we remove the air, we find that all objects, regardless of shape or density, fall with the same acceleration at a particular position near the earth's surface. Furthermore, because $\vec{g}$ does not change direction or magnitude appreciably unless we move through distances comparable with the size of the earth, the acceleration is closely the same for objects falling anywhere within a room, within a building, a city, or even a state. In the region within which the gravitational field $\vec{g}$ is effectively constant and where gravitation alone is important, all objects fall with constant acceleration equal to $\vec{g}$. Starting from rest, in time t they pick up *downward* speed

$$v = gt$$

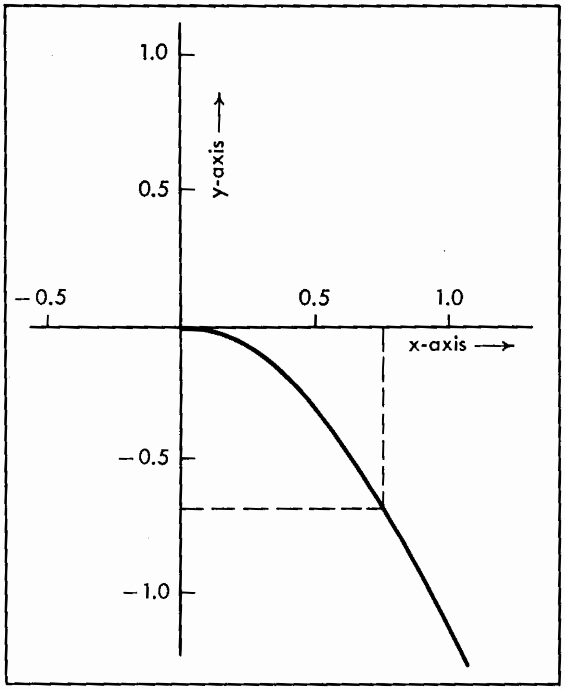
and move down through the distance

$$d = \tfrac{1}{2}gt^2$$

given by the area under the curve of speed versus time (Fig. 21–3). By using the flash photo of Fig. 21–2 again, you can show that the distance d moved from rest increases as $\frac{1}{2}gt^2$.

21–3. Projectile Motion: The Vector Nature of Newton's Law of Motion

Objects that drop straight down under the influence of gravitational attraction alone all accelerate at the same rate. Do all objects also accelerate at that same rate when they move in other directions in the gravitational field? Fig. 21–4 shows a series of flash photographs of two balls. The first ball started falling from rest at the moment that we projected the second one horizontally. We see that the vertical motions of the two balls are identical despite the fact that the horizontal motions differ. Also we see that the horizontal motion is at constant horizontal velocity, like motion when there is no force. The presence of the downward force does not change the horizontal motion; and the existence of horizontal



21–5. The path of the second ball in Fig. 21–4 is plotted on a pair of coordinate axes. The scales on the figure measure the distance along the coordinate axes in meters. The x coordinate of the ball at any time t is $v_0 t$ ($v_0 = 2$ m/sec), and its y coordinate is $-\frac{1}{2}gt^2$. For example, when $t = 0.38$ sec, $x \approx 0.75$ m and $y = -0.7$ m.

motion does not change the effect of the downward force on the vertical motion. Our observation shows that the horizontal and vertical motions are independent. Each one follows from the appropriate component vector of the force: $\vec{F}_v = m\vec{g}$, and the vertical motion is the same as free fall; $\vec{F}_h = 0$, and the horizontal motion is without acceleration.

At the end of the last chapter we found that the net force $\vec{F}$ acting on a body is a vector. As we know from Chapter 6, the acceleration $\vec{a}$ is also a vector. This suggested that Newton's law of motion is a vector law. In addition, when the force $\vec{F}$ acts in the direction of motion, the acceleration $\vec{a}$ is related to the force by $\vec{F} = m\vec{a}$. In the last chapter, however, we did not investigate any motion in which the net force $\vec{F}$ acted at an angle to the velocity $\vec{v}$. We might, therefore, ask the questions: Is Newton's law of motion valid when $\vec{F}$ and $\vec{v}$ are in different directions? Is the acceleration still in the direction of the force? Is the magnitude of $\vec{a}$ the same for the same size force?

Projectile motion is the first case we have examined in which $\vec{F}$ and $\vec{v}$ are in different directions. The observed fact is that the vertical gravitational force $\vec{F}$ produces the same vertical acceleration no matter whether there is horizontal motion or not. Although this observation is not enough to prove that $\vec{F} = m\vec{a}$, it provides supporting evidence for the idea that Newton's law in this simple form holds regardless of the direction of motion. After discussing the path of a projectile, we shall examine experiments with other forces acting across the direction of motion. We shall see that the vector law $\vec{F} = m\vec{a}$ also holds for those experiments.

21–4. Projectile Motion: Determination of the Path

In studying the motion of a projectile we encounter a new problem. Previously in our study of dynamics, we considered objects moving in a straight line under the action of a net force along the line of motion. The second ball in Fig. 21–4, however, traveled in a curved path. One of the important problems in dynamics is the determination of the path in such a situation.

If we know the position and velocity of an object at one moment, the path it follows as a function of time can be found from the force on the object and Newton's law of motion. For the projectile, however, we do not need to go all the way back to Newton's law and the gravitational force. We can find the path by combining the known vertical and horizontal motions. (As we saw in the last section, they go on independently in agreement with Newton's law and the known gravitational force.) For this purpose we choose a horizontal axis of reference (the x axis) and a vertical reference axis (the y axis) so placed that the origin ($x = 0$, $y = 0$) is at the point where the body is projected. (Fig. 21–5.) When the ball is projected with horizontal velocity v_0, we know that it continues to move along the x direction at this velocity. After a time t, the x coordinate of the position of the ball is therefore $x = v_0 t$. We also know that the vertical motion is the same as free fall. The y coordinate at the same time t is therefore $y = -\frac{1}{2}gt^2$. (The minus sign just tells us that the ball goes down rather than up). These equations contain all the information about bodies projected horizontally with an initial velocity equal to v_o. The common value of the time t in the equations relates them to the motion of one single body rather than to the motion of two different bodies.

The path which the body follows is a curve, and we can express this curve by an equation relating the vertical position y to the horizontal position x at the same instant of time. To find this equation we eliminate the time t from the two equations $y = -\frac{1}{2}gt^2$ and $x = v_o t$. From the second equation we see that $t = x/v_o$; and putting this expression for t into the first equation, we get

$$y = -\tfrac{1}{2}gt^2 = -\tfrac{1}{2}g\left(\frac{x}{v_o}\right)^2 = -\frac{g}{2v_o^2}x^2.$$

The equation

$$y = -\frac{gx^2}{2v_o^2}$$

is the equation of the path of the object. As shown in Fig. 21–5, the path is a parabola with its vertex at the place where the object is moving horizontally.

In Fig. 21–6 we have plotted several possible paths which correspond to different values of the initial horizontal velocity v_o. As we see, when the horizontal velocity is large, the parabola is rather flat. The projectile moves a long way sidewise before falling any great distance. On the other hand, small values of v_o give sharply curved parabolas. The projectile moves a shorter distance horizontally in the time it takes to fall a given amount.

Here we have analyzed the problem of a projectile which is fired horizontally. The more general case of a projectile fired with an initial velocity $\vec{v}_o$ at any angle with the horizontal can be handled in the same way. We again use the fact that the vertical and horizontal motions are independent. From the initial velocity vector $\vec{v}_o$, we find the initial horizontal and vertical components. The horizontal component of velocity never changes, and the vertical component undergoes a uniform change at the rate

$$\frac{\Delta v_y}{\Delta t} = -g.$$

On working through the details the result is

21–6. Several possible paths for a body projected horizontally. Note that the shape of the parabola depends on the magnitude of the horizontal velocity v_o.