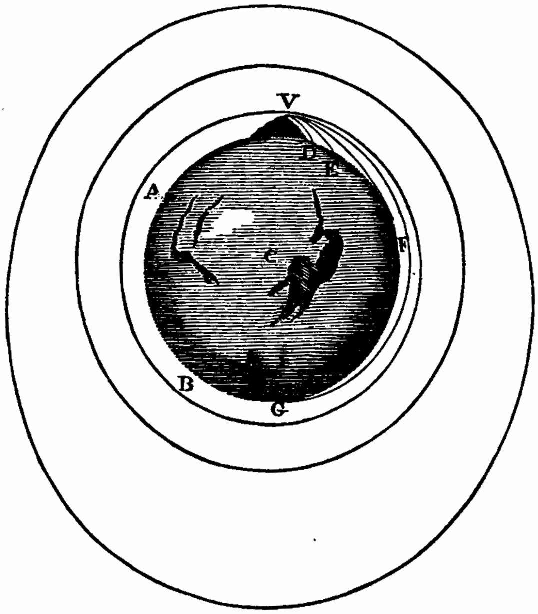


21–12. A drawing from Newton's "System of the World" (attached to later editions of the "Principia"), showing the paths that a body would follow if projected with various speeds from a high mountain. As you can see, Newton was aware that a body would go into an orbit around the earth if its speed was great enough. The orbits from *V* ending at *D, E, F,* and *G* are for projectiles thrown with higher and higher horizontal velocities. Newton recognized that air resistance would prevent the motion of satellites close to the earth from following his ideal paths or from continuing for a long time. He pointed out that satellites could move permanently in the outer orbits.

It is one of the triumphs of the development of the dynamics of Galileo, Newton, and others that it was possible to discuss such projects with confidence in their ultimate possibility.

Why, then, was the first artificial satellite not placed in orbit in the seventeenth century? You can guess the answer. There were no guns or rockets powerful enough. Man's understanding of broad scientific laws often runs ahead of technology. The detailed application of scientific knowledge requires much time and labor.

Sometimes it is the other way round — technology runs ahead of science. Now that our technology has enabled us to establish artificial satellites, we will gain new observations of the universe. Some of these observations have previously been denied us by the curtain of the earth's atmosphere. The new data will give us new knowledge of cosmic rays and of the density of matter in interplanetary space, and will help shape our ideas of the universe. In the long run basic knowledge and technological applications go hand in hand — one helps the other.

21–7. The Moon's Motion

The moon is an earth satellite. We can compute its centripetal acceleration from the following observations. The period of the moon's motion is 27.3 days, or 2.3×10^6 sec; and the distance from the earth to the moon is about 3.8×10^8 meters (about 240,000 miles). The magnitude of the moon's acceleration toward the earth is

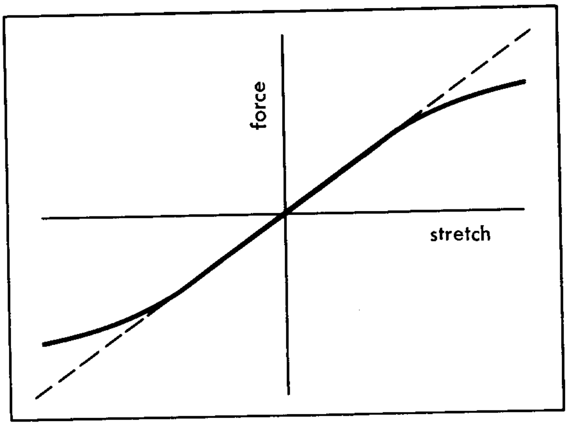
$$a = \frac{4\pi^2 R}{T^2} = \frac{4 \times \pi^2 \times 3.8 \times 10^8\ \text{m}}{(2.3 \times 10^6\ \text{sec})^2}$$
$$= 2.7 \times 10^{-3}\ \text{m/sec}^2.$$

This acceleration is much smaller than the acceleration of a satellite near the earth's surface. Comparing it with $g = 9.8$ m/sec², we see that the gravitational attraction has fallen off by a factor of about 2.7×10^{-4}. This evidence of the weakening of gravitational attraction as the separation increases was one of the things that led Newton to his law of gravitation, as we shall see in the next chapter.

21–8. Simple Harmonic Motion

When we stretch a spring, it pulls back with a force proportional to the stretch (Fig. 21–13). If we attach a mass to a stretched spring and let it go, it will oscillate to and fro. We often find such forces which are proportional to the distortion, at least for small distances. We can describe them by $\vec{F} = -k\vec{x}$, where $\vec{x}$ is essentially the distortion or stretch distance, and the minus sign indicates that the force is a restoring force which pulls the system back toward its equilibrium position.

Linear restoring forces like these always lead to similar to-and-fro motions, called simple harmonic motion, and we want to know what this motion looks like. If we were to use Newton's law directly to predict the motion, we would be confronted with a mathematically complicated problem; but the motion can be derived from circular motion, as we shall now show. Circular motion in a plane looks just like the simple harmonic to-and-fro motion if we examine it from the edge of the plane. To get an idea of



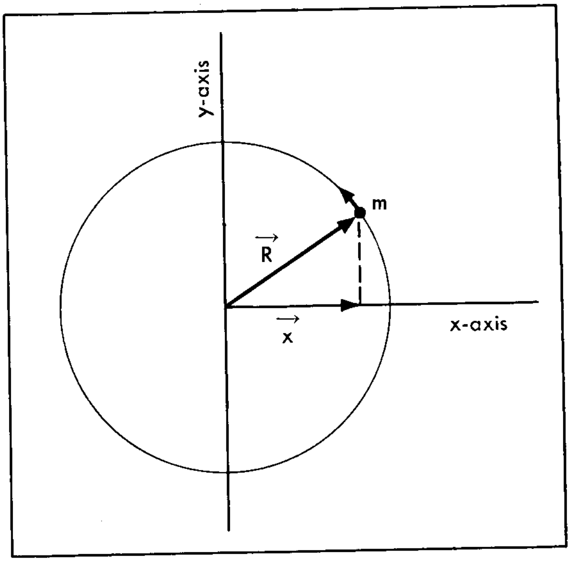
21–13. The force exerted by a spring as a function of its stretch, or of its compression. When the distortion of the spring is not too great, the force is directly proportional to the distortion. The curve in this region can be approximated by a straight line, the ends of which are shown dashed.

the motion, move your thumb steadily around a horizontal circle level with your eye. The straight-line motion you see is simple harmonic motion, as we can tell by matching such a motion with the motion of a mass on a spring (Fig. 21–14).

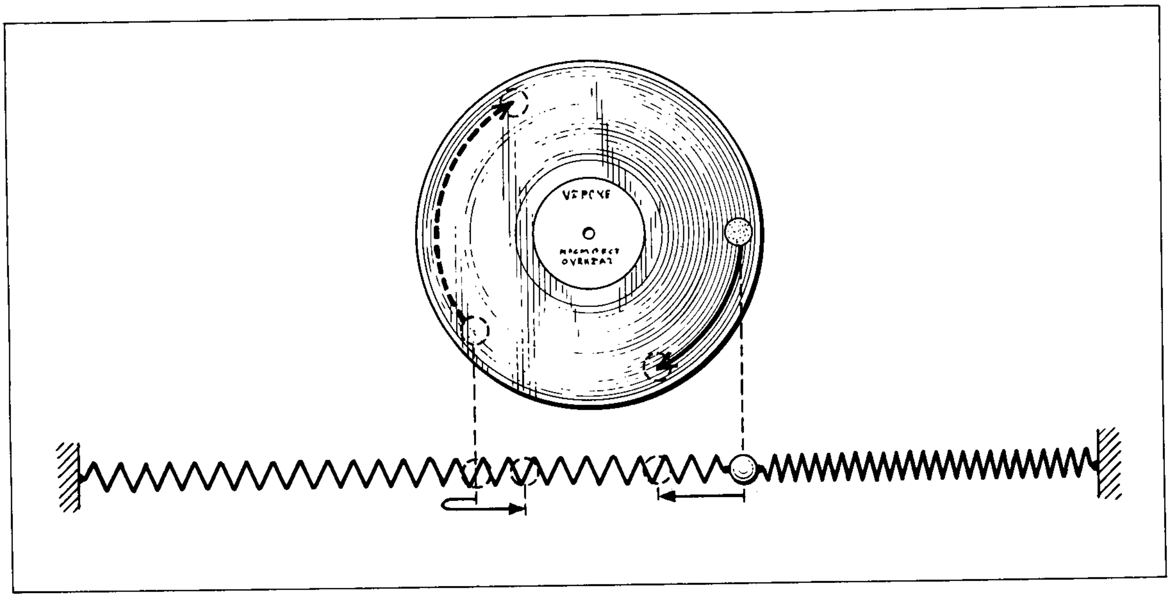
To study this motion we once again return to the fundamental property of vectors which allows us to represent any motion in a plane as the combination of two independent motions in fixed directions at right angles to each other. We shall apply this independence to the components of circular motion with constant speed. We shall find the motion of one component and we shall also find the force which gives the motion in that direction.

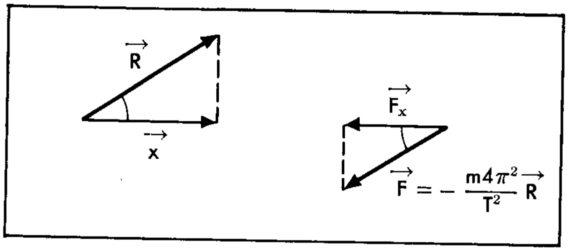
Fig. 21–15 shows a circle around which a body of mass m is moving with constant speed; we shall examine the horizontal component of this motion. In Fig. 21–16 we have drawn a right triangle with the position vector $\vec{R}$ and its vector component $\vec{x}$ in the horizontal direction. We have also drawn

21–14. The to-and-fro motion of a mass on a spring can be matched by one component of the motion of a mass on a circular turntable. We must choose an appropriate radius for the turntable and turn it at the right speed.



21–15. A mass m is moving around a circle at constant speed. This motion can be thought of as consisting of two components at right angles — a horizontal and a vertical component. We shall look closely at the horizontal x component.





21–16. A right triangle is formed by the position vector $\vec{R}$ and its horizontal vector component $\vec{x}$. The centripetal force $\vec{F}$ and its horizontal component $\vec{F}_x$ form another right triangle. The triangles are similar because the angle between $\vec{R}$ and $\vec{x}$ is equal to the angle between $\vec{F}$ and $\vec{F}_x$ ($\vec{R}$ is parallel to $\vec{F}$, and $\vec{x}$ is parallel to $\vec{F}_x$).

the similar right triangle with the centripetal force $\vec{F}$ and its vector component $\vec{F}_x$. From these similar triangles we see that the magnitudes of $\vec{F}_x$ and $\vec{x}$ are in the same ratio as those of $\vec{F}$ and $\vec{R}$ and they point in opposite directions.

Since $\vec{F} = -\frac{m4\pi^2}{T^2}\vec{R}$ (see Section 21–5), the relation between $\vec{x}$ and $\vec{F}_x$ is therefore

$$\vec{F}_x = -\frac{m4\pi^2}{T^2}\vec{x}.$$

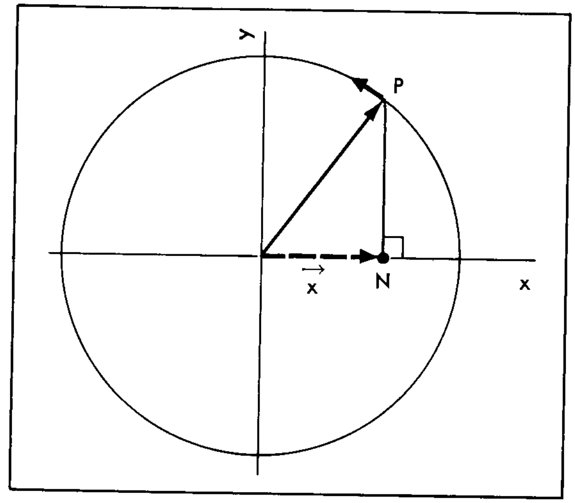
(This equation is simply the x component of the equation $\vec{F} = -\frac{m4\pi^2}{T^2}\vec{R}$.)

Since the mass m and the period T are constant for any particular motion, $m4\pi^2/T^2$ is a constant, and the equation can be more simply written

$$\vec{F}_x = -k\vec{x},$$

where the factor $k = m4\pi^2/T^2$ is the constant of proportionality between $\vec{F}$ and $\vec{x}$. This shows that the force $\vec{F}_x = -k\vec{x}$ is the force that produces the to-and-fro motion of $\vec{x}$. Because the motion along the x axis depends only on the force along the x direction, any such force produces a motion exactly like the motion of point N at the end of $\vec{x}$ in Fig. 21–17 when the point P in the figure goes steadily around the circle.

We have now linked up the x component of circular motion with the motion of any mass m when acted on by a restoring force $\vec{F} = -k\vec{x}$ no matter what provides the force, and this link enables us to calculate the period. If we have a mass m moving under a force $\vec{F} = -k\vec{x}$, we can always imagine a suitable circular motion that will match the motion of m. The period of the actual motion of m is the same as that of this matching circular motion.



21–17. If a mass at N is acted on by a force such that $\vec{F} = -k\vec{x}$, it will move along the x axis. Its motion will be the same as the x component of the motion of a point P moving uniformly in a circle.

We can use our earlier discussion to find the relation between the force constant k in $\vec{F} = -k\vec{x}$ and the period in which the mass moves to and fro. The period depends only on the mass m and on the force constant k. By rearranging $k = \frac{m4\pi^2}{T^2}$, we find

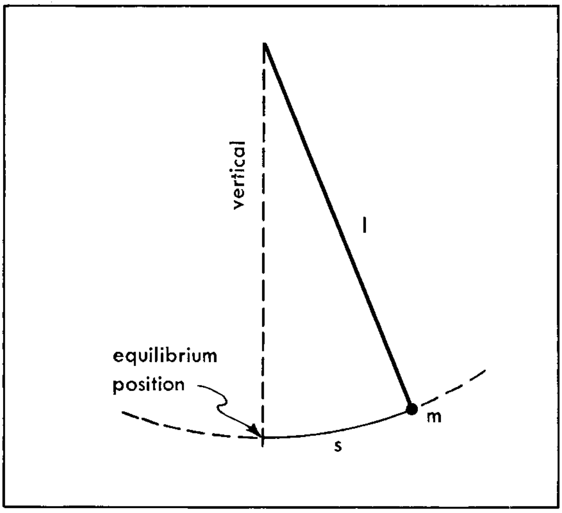
$$T = 2\pi\sqrt{\frac{m}{k}}.$$

This expression is reasonable. If the restoring force increases rapidly with distance (that is, if k is large), the mass is pushed back and forth rapidly: T becomes small. On the other hand, if the mass is large it responds more sluggishly to the force; and so the period is greater the larger the mass. One result of our reasoning may seem astonishing: the period does *not* depend on the amplitude R of the motion. This is a result you can easily check experimentally. Try it.

A simple pendulum is just a mass m on the end of a string of length l. Its motion is very close to simple harmonic motion. To show this, we start by finding the force along the path s (Fig. 21–18); this force shoves the mass m back toward its equilibrium position. As we see in Fig. 21–19, the magnitude of the force F_s along the path is given by

$$\frac{F_s}{mg} = \frac{d}{l} \quad \text{or} \quad F_s = \frac{mg}{l}d,$$

where d is the horizontal displacement of m from equilibrium. There is almost no difference be-



21–18. A simple pendulum: a mass m on a string of length l moves along the path s.

tween the lengths or directions of d and s (Fig. 21–20); we therefore see that F_s is also closely proportional to s with the proportionality factor mg/l. Also, F_s always points back toward the central position, and so we have a linear restoring force:

$$F_s = -\frac{mg}{l}\,s.$$

This force has the form $F_s = -ks$; and for any particular pendulum the constant k is given by the mass m and length l of the pendulum. From

$$k = \frac{mg}{l}$$

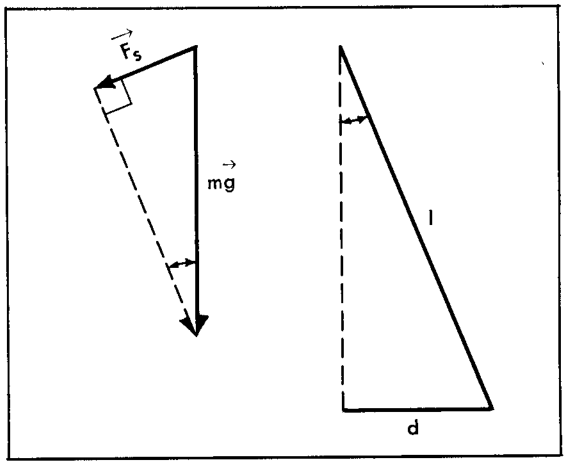
we can find the period of the pendulum. We substitute this expression for k in

$$T = 2\pi\sqrt{\frac{m}{k}}.$$

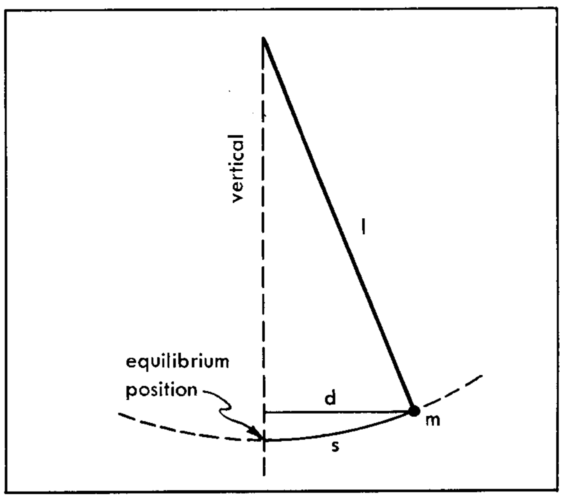
The result is

$$T = 2\pi\sqrt{\frac{l}{g}}.$$

The period depends only on the length of the pendulum and the gravitational field strength. As always when the force is basically gravitational, the mass does not matter. The amplitude or size of the swing also does not matter as long as it is so small that d and s are closely the same. A pendulum of accurately known length and a good independent clock like the rotating earth can thus be used to measure g accurately.



21–19. Two similar triangles. In the left-hand triangle we show $\vec{F}_s$ acting along the path, the weight $\vec{mg}$ acting vertically downward, and the pull on the string (dashed line). The other triangle shows the string of length l, d, the horizontal distance of m from the center line, and the center line. These triangles are similar because $\vec{mg}$ and the center line are both vertical, and the pull on the string, which is perpendicular to $\vec{F}_s$, must therefore be in the same direction as l.



21–20. When the mass is displaced only a short distance from its equilibrium position, there is very little difference between the length of the path s and the horizontal displacement d. Imagine that the displacement is much smaller than it is shown here, compared to the length of the pendulum.

21–9. Experimental Frames of Reference

Turn around. Apparently the room rotates the other way. The corner of the room goes around a circle; so does a marble on the table. But no force mv^2/r on the marble is required to accelerate the marble while it moves in its apparently circular path. Under the action of no net force the marble is simply at rest when we describe it with